Sydney Technical High School

Trial HSC Certificate
Mathematics
2012

Name ....

Teacher ....

General Instructions

- *Reading time – 5 minutes*
- *Working time – 3 hours*
- *Write using black or blue pen Black pen is preferred*
- *Board-approved calculators may be used*
- *A table of standard integrals is provided at the back of this paper*
- *Show all necessary working in Questions 11–16*

Total marks – 100

Section I *Pages 2–5* ***10 marks***

- *Attempt Questions 1–10*
- *Allow about 15 minutes for this section*

Section II *Pages 6–13* ***90 marks***

Attempt Questions 11–16

Allow about 2 hours 45 minutes for this section

Section I

Total marks (10)

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

$2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ● B ~~●~~ C ○ D ○

1. Rationalize the denominator of $\dfrac{1}{9+\sqrt{2}}$.

(A) $\dfrac{9-\sqrt{2}}{7}$ (B) $\dfrac{9-\sqrt{2}}{79}$

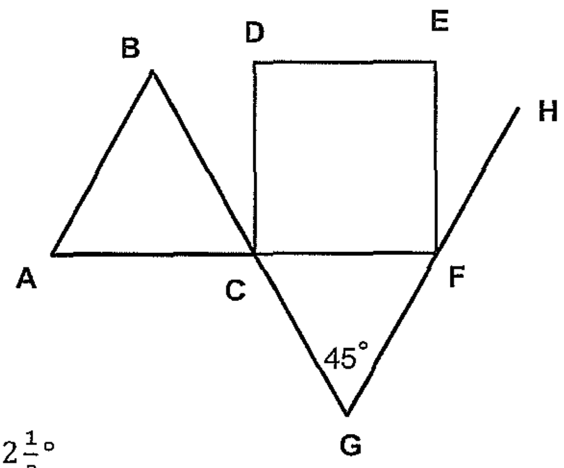
(C) $\dfrac{9+\sqrt{2}}{11}$ (D) $\dfrac{9+\sqrt{2}}{83}$

2. The solution of the equation $3(p-2)=5p+2$ is

(A) $p=-4$ (B) $p=-2$

(C) $p=-1$ (D) $p=1$

3. ABC is an equilateral triangle.
CDEF is s square.



BCG, ACF, and GFH are straight lines.

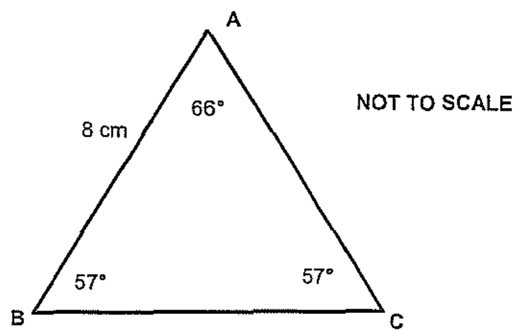
$\angle CGF = 45^\circ$

The size of $\angle EFH$ is

(A) 15° (B) $22\frac{1}{2}$°

(C) 30° (D) 45°

4. The area of ΔABC in square centimetres is closest to.

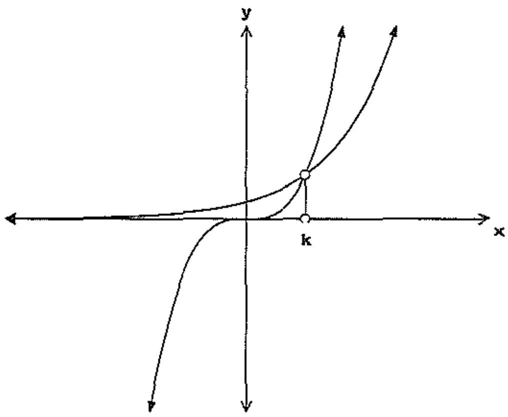


(A) 13.0 (B) 26.8

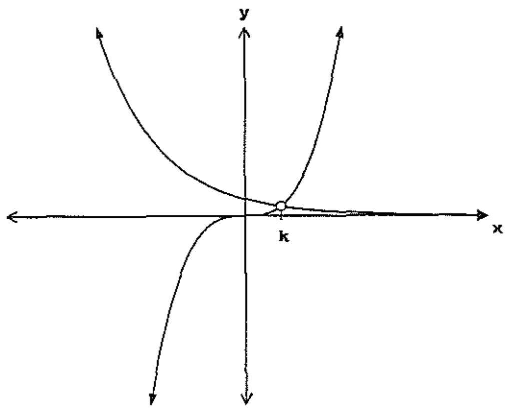
(C) 29.2 (D) 58.5

5. Which of the following could the value of k, be a solution of $2^x - x^3 = 0$

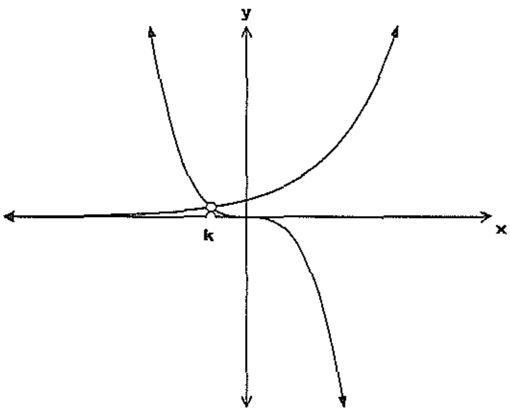
(A)



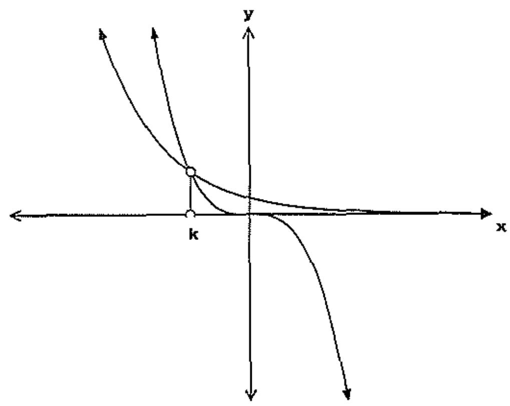
(B)



(C)



(D)



6. Given $y = x(x-5)^2$, find the turnings points and determine their nature.

(A) Minimum at $= \frac{5}{3}$, maximum at $x = 5$
(B) Minimum at $x = -5$, maximum at $x = \frac{3}{5}$
(C) Minimum at $= 5$, maximum at $x = \frac{5}{3}$
(D) Minimum at $= \frac{3}{5}$, maximum at $x = -5$

7. Evaluate $\int_1^4 x^5 + 2x^3 - 6x^2 - 10\,dx$

(A) 654 (B) $642\frac{2}{3}$

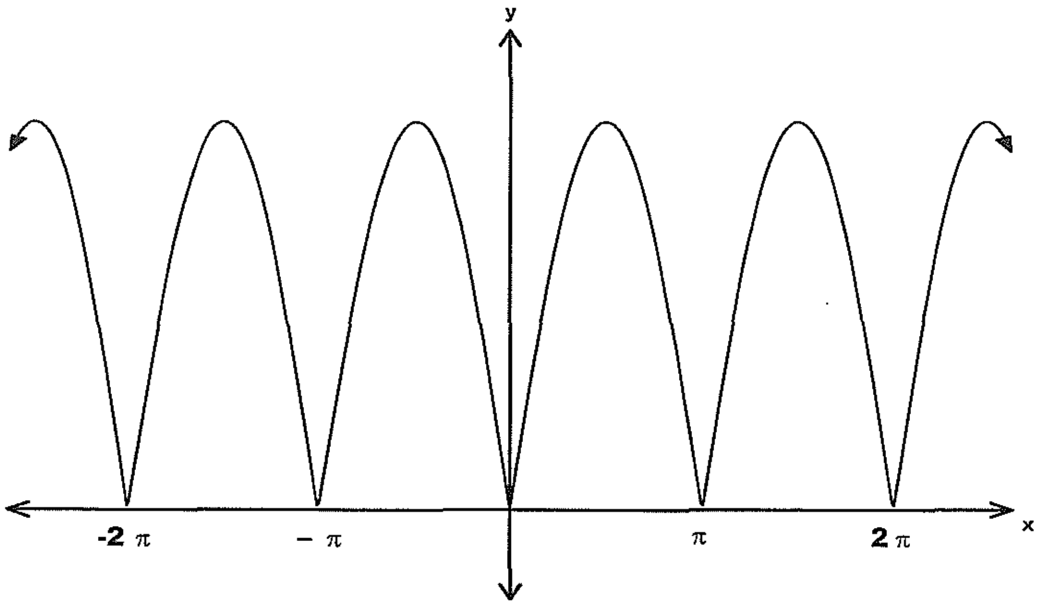
(C) $-11\frac{1}{3}$ (D) $631\frac{1}{3}$

8. Which term of the geometric sequence is 5, 15, 45 … is 885 735?

(A) 10^{th} (B) 11^{th}

(C) 12^{th} (D) 13^{th}

9. The graph below could represent



(A) $y = \sin|x|$ (B) $y = 1 - |\cos x|$

(C) $|y| = |\sin x|$ (D) $y = |\sin x|$

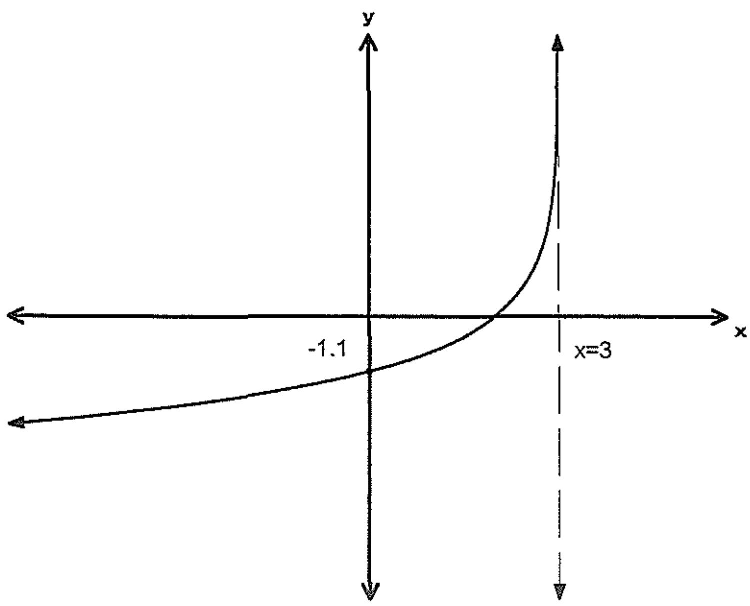
10. The equation of the graph shown could be

(A) $y = log_e(3 - x)$

(B) $y = -log_e(3 - x)$

(C) $y = e^{(x-2)}$

(D) $y = -log_e(3x - 1)$



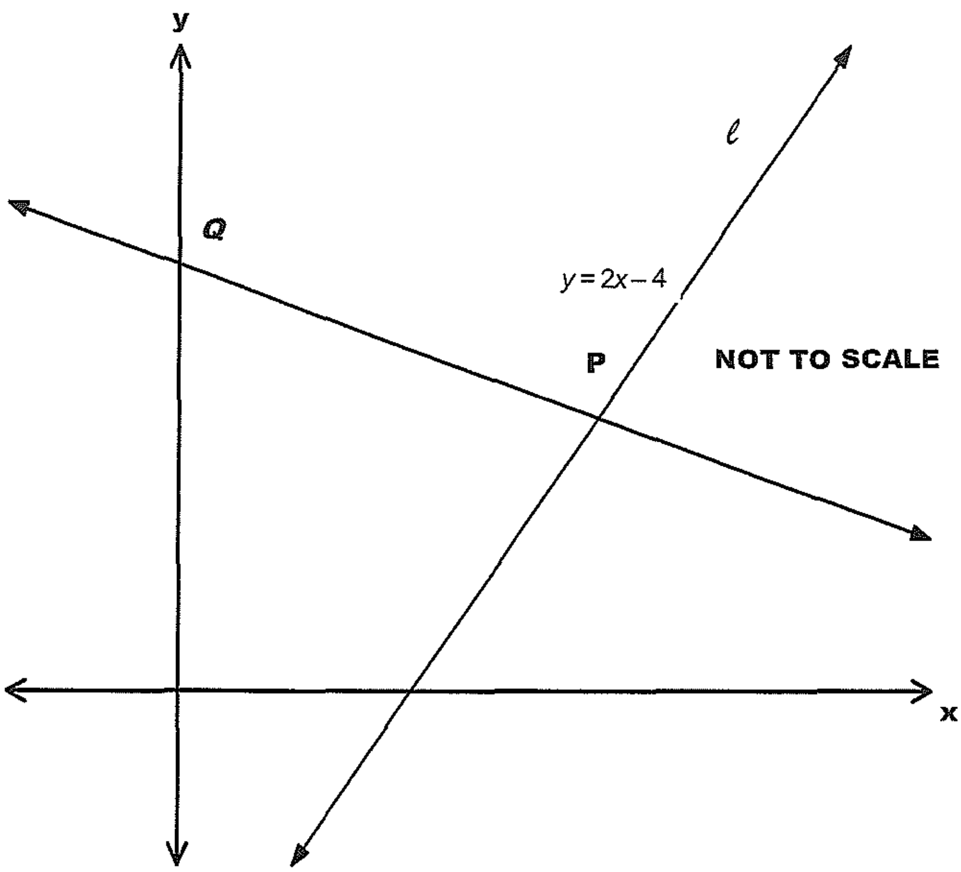
Section II

Total Marks (90)
Attempt Questions 11 - 16
Allow about 2 hours 45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page.

Question 11 **(15 Marks)**

a. The sketch below shows the line PQ and the line (l) $y = 2x - 4$, which is perpendicular to PQ



(i) Show that the point $R(3,2)$ lies on the line l. **1**

(ii) Q is the point (0,5). Find the midpoint of QR. **1**

(iii) Find the equation of the line PQ. **2**

(iv) Find the gradient of QR. **1**

(v) Find the distance QR in simplest surd form. **2**

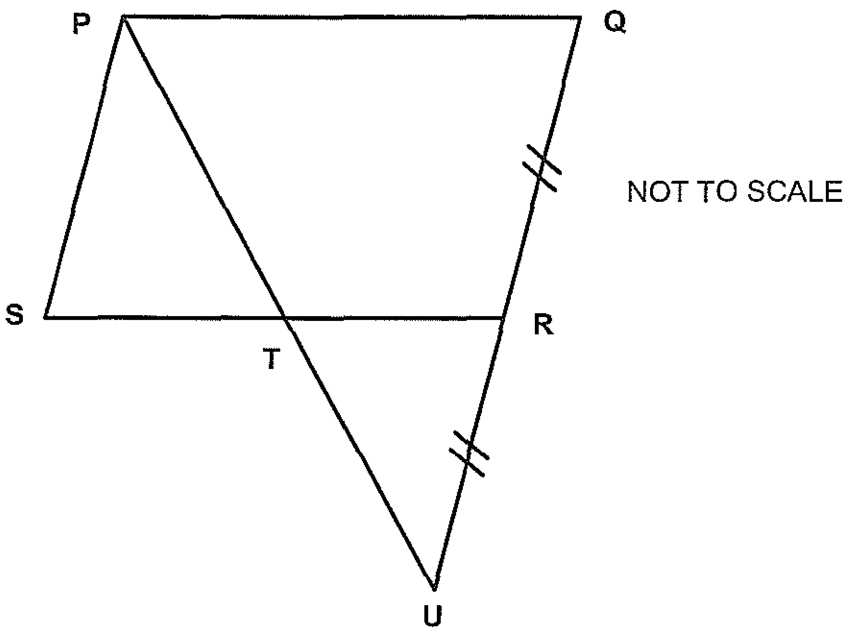
(vi) Find the distance PQ. **2**

b. The nth term of an arithmetic series is given by $T_n = 3n + 4$.

(i) What is the 12^{th} term of the series? **1**

(ii) What is the sum of the first 20 terms of this series? **2**

c. In the diagram, PQRS is a parallelogram. QR is produced to U so that QR=RU.
Copy this diagram into your answer booklet



(i) Giving clear reasons, show that the triangles PST and URT are congruent. **2**

(ii) Hence, or otherwise, show that T is the midpoint of SR. **1**

End of Question 11

Question 12 (START A NEW PAGE) **(15 Marks)**

a. Find the values of q if $3qx^2 - 5x + 3q = 0$ is negative definite. **3**
Leave your answer in exact form.

b. Prove that **3**

$$\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\,cosec\,\theta$$

c. Differentiate:

(i) $\tan 2x$ **1**

(ii) $x^2 \ln x$ **2**

d. Find:

(i) $\int \frac{\cos x}{1+\sin x}\,dx$ **1**

(ii) $\int_0^{\frac{2\pi}{3}} \sin\frac{x}{2}\,dx$ **2**

e. Given that $\frac{d^2y}{dx^2} = \frac{2}{x^2} + 2e^{2x}$ and that when $x = 1$, $\frac{dy}{dx} = e^2$ and $y = \frac{e^2}{2}$,
find an expression for y in terms of x with no other unknown values. **3**

End of Question 12

Question 13 (START A NEW PAGE) **(15 Marks)**

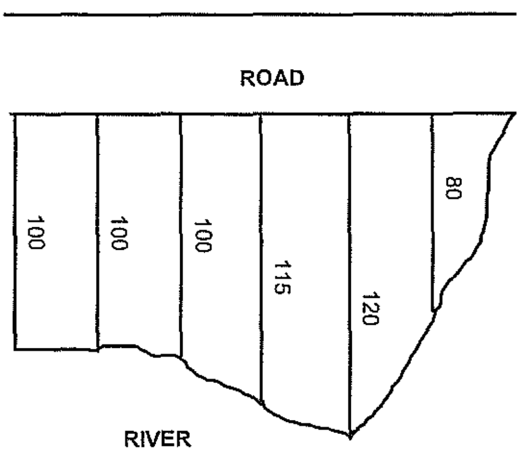
a. The graphs of $y = 5x - x^2$ and $y = x^2 - 3x$ intersect at the origin and at point A.

(i) Show that the co ordinates of A are (4,4) **2**

(ii) Draw a neat sketch of the two graphs on the same number plane. (use a ruler) **1**

(iii) Find the area enclosed by the two graphs. **4**

b. A developer wishes to know the area of land bounded by a straight road, a river and a fence at right angles to the road. A developer hires a surveyor who at 40 metre intervals along the road, measures the distance between the road and the river. Shown below is the surveyor's sketch of this area. All measurements are in metres. **5**



Using Simpson's rule, find the area of the land

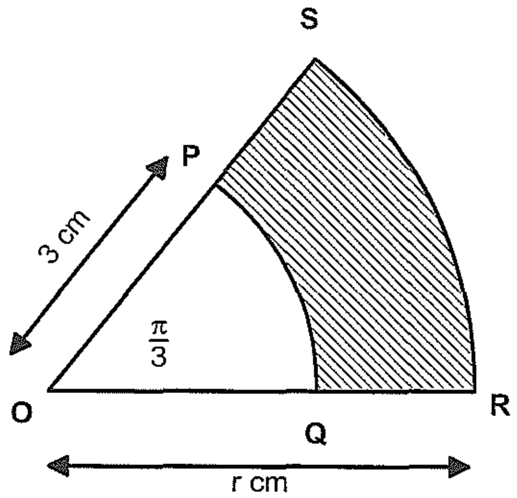
c. $A(-1,3)$ and $B(3,1)$ are two points on the plane. Find the locus of $P(x,y)$ such that PA is perpendicular to PB. **3**

End of Question 13

Question 14 (START A NEW PAGE) **(15 Marks)**

a. In the diagram below PQ and RS are arcs of concentric circles with centre O.

$\angle POQ = \frac{\pi}{3}$ radians and $OP = 3cm.$



(i) Find the area of the sector OPQ **1**

(ii) If OR is r cm, find the area of the sector OSR in terms of r. **2**

(iii) If the shaded area is $\frac{27\pi}{6}$ cm², find the length of PS **2**

b. Consider the curve given by $y = 2 + 3x - x^3$

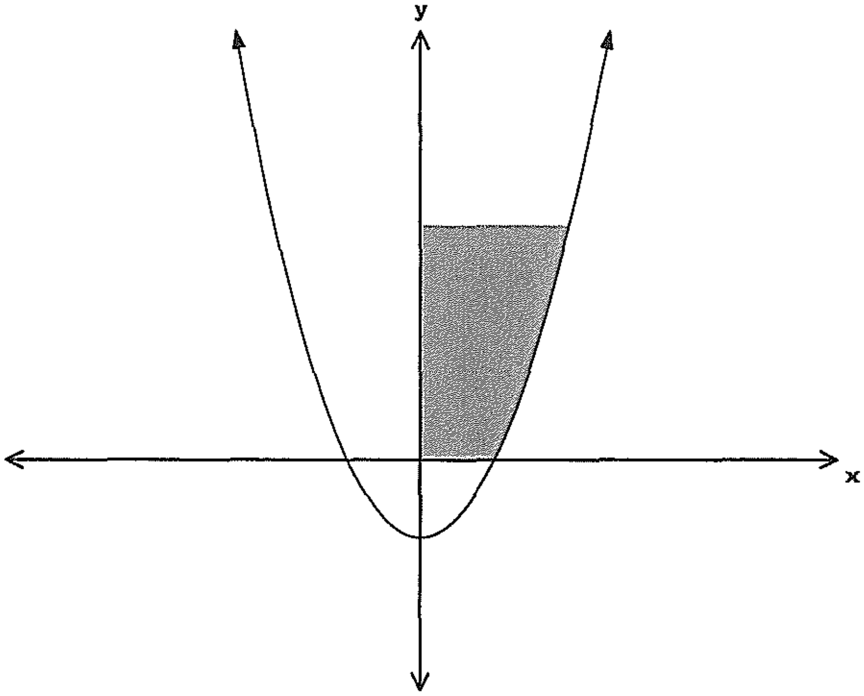
(i) Find $\frac{dy}{dx}$ **1**

(ii) Locate the stationary points and determine their nature **3**

(iii) For what values of x is the curve concave up? **1**

(iv) Sketch the curve, for $-2 \leq x \leq 2$ **2**

c. The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axis. **3**



Find the volume of the solid of revolution when the region is rotated about the y-axis.

End of Question 14

Question 15 (START A NEW PAGE) **(15 Marks)**

a. Solve the following:
$log_2 x + log_2 (x + 7) = 3$ for $x > 0$ **3**

b. Find the equation of the normal to the curve $y = x \, sinx$ at the point where $x = \frac{\pi}{2}$ **3**

c. Consider the function $g(x) = \frac{2}{x^2 - 1}$

(i) Show that g(x) is an even function **1**

(ii) State the domain of $y = g(x)$ **1**

d. Maria is saving for a cruise, She opens an 'Incentive Saver Account' which pays interest at the rate 0.4% per month compounded monthly at the end of each month. Maria decides to deposit \$400 into an account on the first of each month. She makes her first deposit on 1st January 2010 and her last on 1st July 2012.
She withdraws the entire amount, plus interest immediately after her final payment on 31st July 2012.

(i) How much did Maria deposit into her 'Incentive Saver Account'? **1**

(ii) How much did Maria withdraw from her account on 31st July 2012? **3**

(iii) Maria's holiday is cancelled due to illness. She then decides to deposit the amount saved for her holiday, into a different account which offers interest at a rate of 5% p.a. compounded quarterly for 2 years. How much will Maria receive at the end of the investment period. **2**

End of Question 15

Question 16 (START A NEW PAGE) **(15 Marks)**

a. Find the volume generated when the curve $y = \sqrt{\cot x}$ is rotated about the x- axis between $x = \frac{\pi}{3}$ and $x = \frac{\pi}{4}$. Leave your answer in exact form. **3**

b. For the parabola with equation $x^2 = -8y$.

(i) Find the coordinates of the focus (S) of the parabola. **1**

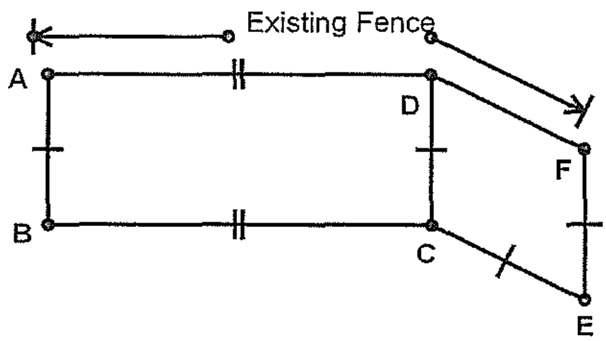
(ii) Find the equation of the directrix of the parabola. **1**

(iii) Show that the point $A(-8, -8)$ lies on the parabola. **1**

(iv) Find the equation of the focal chord of the parabola which passes through A. **2**

(v) Find the equation of the tangent to the parabola at A. **2**

c. A farmer needs to construct two holding paddocks, one rectangular and the other a rhombus for horses and cattle respectively. The diagram below shows the aerial view from directly above the paddocks and how she uses an existing long fence as part of the boundary.



The famer only has 700 m of fencing at her disposal. We also know that $\angle CDF = 30^\circ$
By letting $AB = x$, prove:

(i) A, the area of the paddocks is given by $A = 700x - \dfrac{7x^2}{2}$ **2**

(ii) Hence, find the maximum area. **3**

END OF EXAMINATION
☺

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Sydney Technical High School
Trial HSC Examination
Mathematics 2012

Multiple Choice Answer Sheet

Name ____________________

Teacher ____________________

Completely fill the response oval representing the most correct answer.

1. A ○ B ● C ○ D ○
2. A ● B ○ C ○ D ○
3. A ● B ○ C ○ D ○
4. A ○ B ○ C ● D ○
5. A ● B ○ C ○ D ○
6. A ○ B ○ C ● D ○
7. A ● B ○ C ○ D ○
8. A ○ B ○ C ● D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ● C ○ D ○

Question 11

a) i) $y = 2x - 4$
when $x = 3$ $\quad y = 2 \times 3 - 4$
$y = 2$
$\therefore$ $R(3,2)$ lies on the line

ii) $Q(0,5)$
$R(3,2)$
mid point of $QR = \left(\frac{0+3}{2}, \frac{5+2}{2}\right)$
$= \left(\frac{3}{2}, \frac{7}{2}\right)$

iii) Gradient of $L = 2$
Gradient of line $PQ = -\frac{1}{2}$
Equation of line PQ = $y - 5 = -\frac{1}{2}(x - 0)$
$2y - 10 = -x$
$x + 2y - 10 = 0$.

iv) Gradient of $QR = \frac{5-2}{0-3} = \frac{3}{-3} = -1$.

v) Distance of QR.
$Q(0,5)$
$R(3,2)$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{(3-0)^2 + (5-2)^2}$
$d = \sqrt{9+9}$
$d = \sqrt{18}$
$d = 3\sqrt{2}$ units

vi) $2x - y - 4 = 0$
$(0,5)$
perpendicular distance $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$
$= \left|\frac{2 \times 0 + 1 \times 5 - 4}{\sqrt{2^2 + 1^2}}\right|$

$= \frac{1}{\sqrt{5}}$ or $\frac{\sqrt{5}}{5}$ units b i) $T_n = 3n + 4$ $\quad a = 7$

$T_{12} = 3 \times 12 + 4$ $\quad d = 3$

$T_{12} = 40$

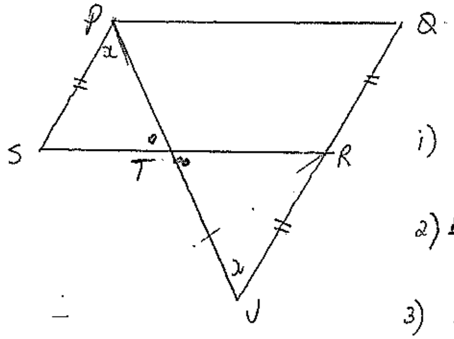
ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2 \times 7 + (20-1)\ 3]$

$= 10[14 + 19 \times 3]$

$= 710$

c i)



In ΔPST & ΔRTU

$RU = QR$ (given)

1) $PS = RU$ (opposite sides of a parallelogram are equal $QR = UR$)

2) $\angle PTS = \angle RTU$ (vertically opposite angles are equal)

3) $PS \parallel QU$

$\angle SPT = TUR$ (alternate angles are equal on parallel lines $PS \parallel QU$)

$\therefore \Delta PST \equiv \Delta RTU$ (AAS)

ii) Since $ST = TR$ (corresponding sides of congruent triangles are equal)

Question 12.

a) $3qx^2 - 5x + 3q = 0$

negative definite $\quad a < 0$

$\Delta < 0$

$\Delta = b^2 - 4ac$

$(5)^2 - 4 \times (3q) \times 3q < 0$

$25 - 36q^2 < 0$

$(5 - 6q)(5 + 6q) < 0$

$q = \pm \frac{5}{6}$

Since $3qx^2 - 5x + 3q$ is negative definite

$q \leq -\frac{5}{6}$

b) $\frac{\sin\theta}{1 - \cos\theta} + \frac{\sin\theta}{1 + \cos\theta} = 2\operatorname{cosec}\theta$

$L.H.S = \frac{\sin\theta}{1 - \cos\theta} + \frac{\sin\theta}{1 + \cos\theta}$

$= \frac{\sin\theta(1 + \cos\theta) + \sin\theta(1 - \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)}$

$= \frac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos\theta}{1 - \cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta}$

$= \frac{2}{\sin\theta}$

$= 2\operatorname{cosec}\theta$

$= R.H.S$

c i) $y = \tan 2x$

$$\frac{dy}{dx} = 2\sec^2 2x$$

ii) $y = x^2 \ln x$

$u = x^2$ $\quad v = \ln x$

$du = 2x$ $\quad dv = \frac{1}{x}$

$$\frac{dy}{dx} = u\,dv + v\,du$$

$$= x^2 \times \frac{1}{x} + \ln x \times 2x$$

$$= x + 2x\ln x$$

d i) $\displaystyle\int \frac{\cos x}{1+\sin x}\,dx = \ln(1+\sin x) + c.$

ii) $\displaystyle\int_0^{2\pi/3} \sin\frac{x}{2}\,dx = \left[-2\cos\frac{x}{2}\right]_0^{2\pi/3}$

$$= \left(-2\cos\left(\frac{1}{2}\times\frac{2\pi}{3}\right)\right) - \left(-2\cos\left(\frac{1}{2}\times 0\right)\right)$$

$$= -1 - (-2)$$

$$= 1$$

e) $\displaystyle\frac{d^2y}{dx^2} = \frac{2}{x^2} + 2e^{2x} \rightarrow 2x^{-2} + 2e^{2x}$

$$\frac{dy}{dx} = \frac{2x^{-1}}{-1} + e^{2x} + c$$

$$= -2x^{-1} + e^{2x} + c$$

when $x = 1$ $\quad \frac{dy}{dx} = e^2$

$e^2 = -2 \times 1^{-1} + e^2 + c.$

$e^2 = -2 + e^2 + c$

$c = 2$

$$\frac{dy}{dx} = -2x^{-1} + e^{2x} + 2$$

$$y = 2\ln x + \frac{1}{2}e^{2x} + 2x + c$$

when $x = 1$ $\quad y = \frac{e^2}{2}$

$\frac{e^2}{2} = 2\ln(1) + \frac{1}{2}e^2 + 2 + c$

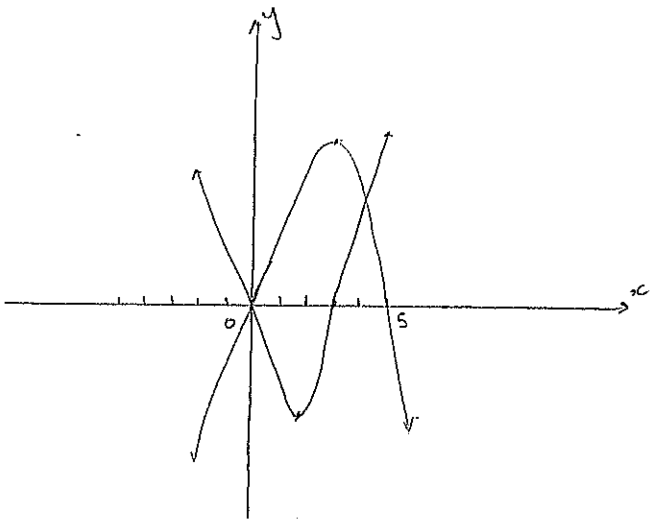
$\frac{e^2}{2} = 0 + \frac{1}{2}e^2 + 2 + c$

$c = -2$

$$\therefore y = 2\ln x + \frac{1}{2}e^{2x} + 2x - 2$$

Question 13

a $y = 5x - x^2$
$y = x^2 - 3x$



$5x - x^2 = x^2 - 3x$
$0 = 2x^2 - 8x$
$0 = 2x(x-4)$

$\begin{cases} x = 0 \\ y = 0 \end{cases} \qquad \begin{cases} x = 4 \\ y = 4 \end{cases}$

$\therefore$ Point of Intersection is $(4, 4)$

$$\int_0^4 5x - x^2 \, dx - \int_0^4 x^2 - 3x \, dx = \int_0^4 2x^2 - 8x$$

$$= \left[\frac{2x^3}{3} - \frac{8x^2}{2} \right]_0^4$$

$$= \left| \left(\frac{128}{3} - \frac{128}{2} \right) - (0) \right|$$

$$= 21\tfrac{1}{3} \text{ units}^2$$

b) $= \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$= \frac{40}{3}\left[(100 + 0) + 4(100 + 115 + 80) + 2(100 + 120)\right]$

$= \frac{40}{3}\left[100 + 1180 + 440\right]$

$= 22\,933\tfrac{1}{3} \text{ m}^2$

c. $PA \perp PB$
$m_1 \times m_2 = -1$

$m_{PA} = \frac{y-3}{x+1}$

i. $m_{PB} = \frac{y-1}{x-3}$

$\frac{y-3}{x+1} \times \frac{y-1}{x-3} = -1$

$(y-3)(y-1) = -(x+1)(x-3)$
$y^2 - y - 3y + 3 = -(x^2 - 2x - 3)$
$y^2 - 4y + 3 = -x^2 + 2x + 3$
$x^2 + y^2 - 2x - 4y = 0$

$(x-1)^2 + (y-2)^2 = 5$
centre $(1, 2)$ radius $= \sqrt{5}$

Question 14

a i) Area of sector OPQ $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 3^2 \times \frac{\pi}{3}$

$= \frac{3\pi}{2}$

ii) Area of sector OSR $= \frac{1}{2} \times r^2 \times \frac{\pi}{3}$

$= \frac{\pi r^2}{6}$

iii) $\frac{27\pi}{6} = \frac{\pi r^2}{6} - \frac{3\pi}{2}$

$6\pi = \frac{\pi r^2}{6}$

$36\pi = \pi r^2$

$r^2 = 36$

$r = 6$

PS = 3 cm

b i) $y = 2 + 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2$ $\frac{d^2y}{dx^2} = -6x$

ii) Stationary points occur when $\frac{dy}{dx} = 0$.

$3 - 3x^2 = 0$

$3(1 - x^2) = 0$

$(1-x)(1+x) = 0$ $x = 1$ $x = -1$

$y = 4$ $y = 0$

When $x = 1$

$\frac{d^2y}{dx^2} = -6 < 0$

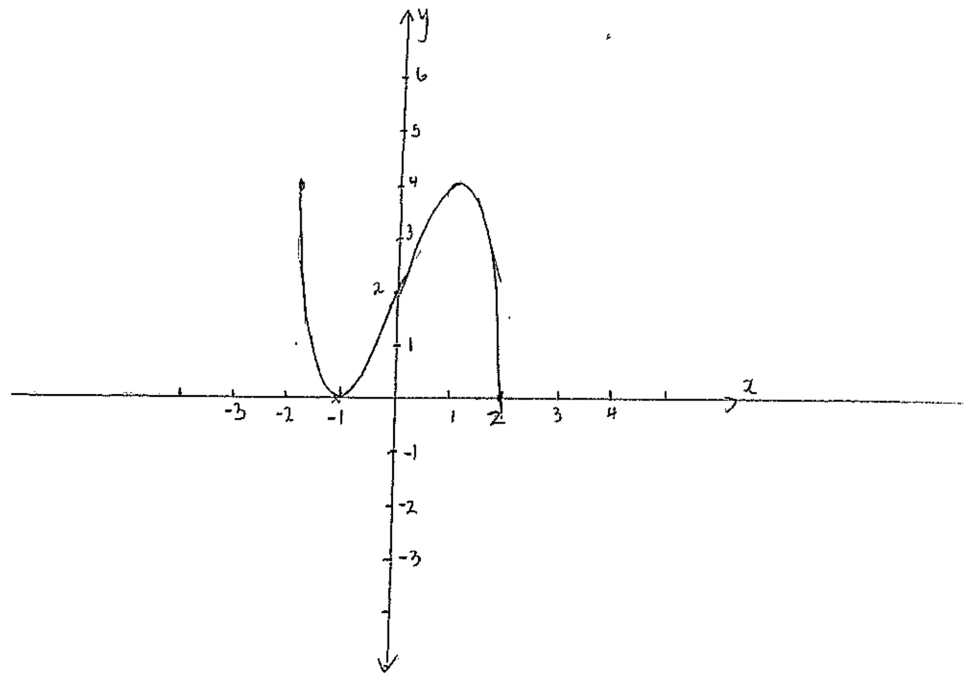
$\therefore$ maximum occurs at $(1, 4)$

When $x = -1$

$\frac{d^2y}{dx^2} = 1 > 0$

$\therefore$ minimum occurs at $(-1, 0)$

b iii) concave up $x < 0$.



c)

$$y = 2x^2 - 2$$

$$y + 2 = 2x^2$$

$$\frac{y}{2} + 1 = x^2$$

$$V = \pi \int_a^b x^2 \, dy$$

$$V = \pi \int_0^6 \frac{y}{2} + 1 \, dy$$

$$= \pi \left[\frac{y^2}{4} + y \right]_0^6$$

$$= \pi \left[\frac{36}{4} + 6 \right] - \left[0 \right]$$

$$= 15\pi \text{ units}^3.$$

Question 15

a) $\log_2 x + \log_2 (x+7) = 3$ $\quad\quad$ $\underline{x > 0}$

$$\log_2 \left[x(x+7) \right] = 3.$$

$$\left[x(x+7) \right] = 2^3$$

$$x(x+7) = 8$$

$$x^2 + 7x - 8 = 0$$

$$(x+8)(x-1) = 0$$

$x = -8$ $\quad\quad$ $\underline{x = 1}$

No Solution $\quad\quad$ only solution.

b) $y = x \sin x$

$$\frac{dy}{dx} = x \cos x + \sin x$$

when $x = \frac{\pi}{2}$ $\quad\quad$ $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$m_1 = 1$

$m_2 = -1$

$\therefore$ Equation of the normal

$$y - \frac{\pi}{2} = -1\left(x - \frac{\pi}{2}\right)$$

$$y - \frac{\pi}{2} = -x + \frac{\pi}{2}$$

$$x + y - \pi = 0$$

c) $g(x) = \frac{2}{x^2-1}$ $\qquad \frac{2}{(x-1)(x+1)}$

$g(-x) = \frac{2}{(-x)^2-1}$

$= \frac{2}{(x)^2-1}$

$\therefore g(x) = g(-x)$

Domain: all real for x $\quad x \neq \pm 1$

d) $r = 0.4\%$
$m = 400.$
$n = 30.$

i) $400 \times 31 = \$12\,400.$

ii) $= 400(1.004) + 400(1.004)^2 + 400(1.004)^3 + \ldots + 400(1.00$

$= 400\left[1.004 + 1.004^2 + 1.004^3 + \ldots + 1.004^{31}\right]$

$= 400 \times \frac{1.004(1.004^{31}-1)}{1.004-1}$

$= \$13\,226.29$

iii) $P = 10000$
$r = 5\% \div 4$
$= 0.0125$
$n = 2 \times 4$
$= 8$

$A = 13\,226.29(1.0125)^8$
$= \$14\,608.25$

Question 16

a) $y = \sqrt{\cot x}$
$y^2 = \cot x$

$V = \pi \int_a^b y^2 \, dx$

$V = \int_{\pi/4}^{\pi/3} \cot x \, dx$

$V = \pi \int_{\pi/4}^{\pi/3} \frac{\cos x}{\sin x} \, dx.$

$V = \pi \left[\ln(\sin x)\right]_{\pi/4}^{\pi/3}$

$V = \pi \left[\ln \frac{\sqrt{3}}{2} - \ln \frac{1}{\sqrt{2}}\right]$

$V = \pi \ln\left(\frac{\sqrt{6}}{2}\right)$ units^3.

b) i) $x^2 = -8y$
$x^2 = -4(2)y$
$a = 2.$
Focus $(0, -2)$

ii) Directrix $y = 2$

iii) $x^2 = -8y$
when $x = 8$ $\quad y = -8$
$x^2 = 8^2 = 64$
$-8 \times y = -8 \times -8 = 64$
$\therefore (8, -8)$ does lies on the parabola

iv) Focal chord $(0,-2)$ $(-8,-8)$

$$\frac{y+2}{x-0} = \frac{-8--2}{-8-0}$$

$$\frac{y+2}{x} = \frac{-6}{-8}$$

$$-8(y+2) = -6x$$
$$-4(y+2) = -3x$$
$$-4y-8 = -3x$$
$$3x-4y-8 = 0.$$

v.) $x^2 = -8y$

$$y = \frac{-x^2}{8}$$

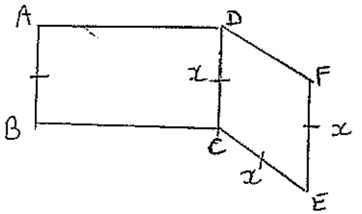
$$\frac{dy}{dx} = \frac{-2x}{8} \rightarrow -\frac{x}{4}$$

$(-8,-8)$

$m = 2$

$$y+8 = 2(x+8)$$
$$y+8 = 2x+16$$
$$y = 2x+8$$

)



let $AB = x$

$AB + DC + CE + EF + BC = 700$

$x + x + x + x + BC = 700$

$4x + BC = 700$

$BC = 700 - 4x$

$$\text{Area} = x(700-4x) + 2\left(\tfrac{1}{2} \times x^2 \times \sin 30\right)$$
$$= x(700-4x) + 2\left(\tfrac{1}{2} \times x^2 \times \tfrac{1}{2}\right)$$
$$= 700x - 4x^2 + \frac{x^2}{2}$$
$$A = 700x - \frac{7x^2}{2}$$

$$A = 700x - \frac{7x^2}{2}$$

$$\frac{dA}{dx} = 700 - 7x$$

$$\frac{d^2A}{dx^2} = -7$$

Stationary points occur when $\frac{dA}{dx} = 0$

$$700 - 7x = 0$$
$$700 = 7x$$
$$x = \frac{700}{7}$$

$$x = 100$$
$$A = 700 \times 100 - \frac{7(100)^2}{2}$$
$$= 35\,000$$

Since $\frac{d^2A}{dx^2} < 0$ for all x $x = 100$ will give the maximum area.